

Influence of a polaron dispersion and excitonic effect on a magnetopolaron energy spectrum in a quantum well

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It has been shown, that a magnetopolaron discrete energy spectrum is realized in the optical experiments where exciting light is directed perpendicularly to a quantum well (QW) plane. The phonon dispersion as well as Coulomb attraction can only shift the magnetopolaron discrete energy levels and broad some of them.

I. INTRODUCTION.

Let us consider an energy spectrum of the electronic excitations in a QW in a strong magnetic field directed perpendicularly to the QW plane. So long as the system is homogeneous in the QW plane (xy plane) any excitation (an exciton or electron-hole pair) can be characterized by a quasi-momentum $\hbar\mathcal{K}_\perp$ in the QW plane, if the wave functions are chosen properly. Such functions have been obtained earlier for the excitons [1] and magnetopolaron-hole pairs [2]. The quasi-momentum $\hbar\mathcal{K}_\perp$ may be a quantum number because the excitation, consisting of an electron and hole, is neutral one. (Let us remind that in a strong magnetic field there are not the states for an electron (hole) where \mathcal{K}_\perp would be a quantum number). Our aim is to determine in principle how the magnetopolaron theory changes when a LO phonon dispersion and excitonic effect are taken into account, i. e. a Coulomb interaction between an electron, defining the polaron, and hole, weakly interacting with LO phonons.

If the excitations are created in a QW by light, the condition $\mathcal{K}_\perp = \kappa_\perp$ must be satisfied, where κ_\perp is the inplane projection of the light wave vector. One obtains $\mathcal{K}_\perp = 0$ if $\kappa_\perp = 0$. It is obviously that only discrete energy spectrum is possible at $\mathcal{K}_\perp = 0$ and in the case of a finite motion. In particular that means that at a normal light incidence neither the LO phonon dispersion, nor Coulomb interaction between an electron and hole results in a broadening of the energy levels into the bands. The mentioned above factors may only shift the discrete energy levels and change the corresponding inverse lifetimes γ . Our results confirm this general statement. We have shown that the excitonic effect leads to the dependence of the energy of the magnetopolaron-hole pair from the quasi-momentum $\hbar\mathcal{K}_\perp$ of the aggregate motion, what can be detected in experiments including an oblique light incidence on the QW plane.

The magnetopolaron effect (the Johnson-Larsen effect) has been discovered in [3,4,5] (see also the re-

views [6,7,8]). The magnetopolarons are created in 3D-systems as well as in 2D-ones, for instance, in QWs. The distance between the magnetopolaron energy levels in 3D-systems is $\sim \alpha^{2/3}\hbar\omega_{LO}$ [9], where α is the Fröhlich non-dimensional electron-phonon coupling constant [10], and in 2D-systems $\sim \alpha^{1/2}\hbar\omega_{LO}$ [11,12,13,14,15,16,17,18,19,20,21,22,23,24].

II. INFLUENCE OF THE PHONON DISPERSION ON THE MAGNETOPOLARON ENERGY SPECTRUM.

The theory of magnetopolarons in a QW without taking into account a phonon dispersion (i. e. in an approximation, where all the phonons, taking part in a polaron formation, have the same frequency ω_{LO}) have been proposed in [19]. The electrons interact with the confined and interface phonons. In the continuum approximation [25] (i. e. in the limit $a \rightarrow 0$, where a is the lattice constant) the confined phonons have the frequency ω_{LO} without a dispersion, but the interface phonons have a dispersion, i. e. the dependence of the frequency from the modulus q_\perp of the inplane phonon wave vector. The frequency of the interface phonons depends on the parameter $q_\perp d$, where d is the QW's width. One has to take into account the dependence of the phonon frequency from $q_\perp a$. It has been shown in [23], that in the case of the wide QWs the approximation is applicable, in which the interaction with the bulk phonons is substituted for the interaction with the confined phonons, and the interaction with the interface phonons may be neglected. Obviously, that in such a case we have only a dispersion due to the deviation from the continuum model. In [14] the polarons are considered with taking into account the interaction with the interface phonons. In [2,23] a classification of the magnetopolarons has been demonstrated. For example let us consider here the magnetopolaron A. It appears as a result of the crossover of

the energy levels of the electron-phonon system with the indexes $m, n = 0, N = 1$ and $m, n = 1, N = 0$, respectively. m is the size-quantization quantum number, n is the Landau quantum number, N is the number of LO phonons. The energies of the first and second levels are $\varepsilon_m^e + \Sigma_0 + \hbar\omega_{LO}$ and $\varepsilon_m^e + \Sigma_1$, respectively, where ε_m^e is the energy of the m -th size-quantized energy level, measured from the QW's bottom. For instance, for the QW with the infinitely high barriers

$$\varepsilon_m^e = \frac{\hbar^2 \pi^2 m^2}{2m_e d^2}, \quad (1)$$

where m_e is the electron effective mass, $m = 1, 2, 3, \dots$. The designations are introduced

$$\Sigma_0 = \hbar\omega_{eH}/2, \quad \Sigma_1 = 3\hbar\omega_{eH}/2, \quad \omega_{eH} = |e|H/(m_e c), \quad (2)$$

e is the electron charge, H is the magnetic field intensity, c is the light velocity in a vacuum. We do not take the phonon dispersion into account up to now. Obviously the energy levels cross over when

$$\omega_{eH} = \omega_{LO}. \quad (3)$$

When the resonance condition Eq. (3) is satisfied, the role of the electron-phonon interaction increases sharply, what leads to the splitting of the energy levels of the electron-phonon system and to the magnetopolaron formation.

The theory, which has been developed in [19], is applicable when the QW's widths are not too wide and, consequently, the distance between the magnetopolaron levels is small in comparison to the distance between the size-quantized energy levels. It has been shown in [23] that the last condition is satisfied for a QW of the system AlSb/GaAs/AlSb at $d \leq 500\text{\AA}$. The low temperatures are supposed when the optical phonons are not excited.

We consider a rectangular QW of the I type with an energy gap E_g . The magnetic field is directed along the axis z perpendicularly to the QW's plane, the vector-potential is $\mathbf{A} = \mathbf{A}(0, xH, 0)$. The electron wave function in the QW is

$$\Psi_{n,k_y,m}^e(x, y, z) = \Phi_n(x + a_H^2 k_y) \frac{1}{\sqrt{L_y}} e^{ik_y y} \varphi_m^e(z), \quad (4)$$

where

$$\Phi_n(x) = \frac{e^{-x^2/2a_H^2} H_n(x/a_H)}{\sqrt{\pi^{1/2} 2^n n! a_H}}, \quad a_H = \sqrt{\frac{c\hbar}{|e|H}},$$

$H_n(t)$ is the Hermitian polynomial, L_y is the normalization length, $\varphi_m^e(z)$ is the real electron wave function, corresponding to the m -th size-quantized energy level (see, for instance, [19]). The electron-phonon interaction is written as

$$V = \sum_{\nu} [C_{\nu}(\mathbf{r}_{\perp}, z) b_{\nu} + C_{\nu}^*(\mathbf{r}_{\perp}, z) b_{\nu}^{\dagger}], \quad (5)$$

where ν is the set on indexes, consisting of \mathbf{q}_{\perp} and other indexes j , which characterize the confined and interface phonons; b_{ν}^{\dagger} (b_{ν}) is the phonon creation (annihilation) operator,

$$C_{\nu}(\mathbf{r}_{\perp}, z) = C_{\nu} e^{i\mathbf{q}_{\perp} \cdot \mathbf{r}_{\perp}} \eta_{\nu}(z); \quad (6)$$

the values $C_{\nu} \eta_{\nu}(z)$ for the electron-phonon interaction with phonons are determined in [25].

As it has been shown in [23], at $d \geq 200\text{\AA}$ in GaAs the application of the electron-bulk phonon interaction is a good approximation. In this approximation the index set ν transits into \mathbf{q} , where $\mathbf{q} = (\mathbf{q}_{\perp}, q_z)$ is the 3D phonon wave vector and, according to [10],

$$\eta_{\nu}(z) = e^{iq_z z}, \quad C_{\nu} = C_q = -i\hbar\omega_{LO} \left(\frac{4\pi\alpha l^3}{V_0} \right)^{1/2} \frac{1}{ql}, \quad (7)$$

where

$$l = \sqrt{\frac{\hbar}{2m_e\omega_{LO}}}, \quad \alpha = \frac{e^2}{2\hbar\omega_{LO}l} \left(\frac{1}{\varepsilon_{\infty}} - \frac{1}{\varepsilon_0} \right),$$

V_0 is the normalization volume, $\varepsilon_0(\varepsilon_{\infty})$ is the static (high frequency) dielectric function [10]. For GaAs $\alpha \simeq 0.071$, $l \simeq 40\text{\AA}$. Applying Eqs. (4) and (6), one finds that the interaction matrix elements are equal

$$\int d^3r \Psi_{n',k'_y,m}^{e*} C_{\nu}^*(\mathbf{r}_{\perp}, z) \Psi_{n,k_y,m}^e = U_{n,n'}^*(\nu) \times e^{(ia_H^2 q_x (k_y + k'_y)/2)} \delta_{k'_y, k_y - q_y}, \quad (8)$$

where the designations are introduced:

$$U_{n,n'}^*(\nu) = C_{\nu}^* K_{n,n'}(a_H \mathbf{q}_{\perp} \times \mathbf{H}/H) \mathcal{M}^*(\nu), \quad (9)$$

$$K_{n,n'}(\mathbf{s}) = \sqrt{\frac{\min(n!, n')!}{\max(n!, n')!}} i^{|n-n'|} \left(\frac{s}{\sqrt{2}} \right)^{|n-n'|} \times \exp\left(-\frac{s^2}{4}\right) \exp[i(\phi - \pi/2)(n - n')] L_{\min(n,n')}^{|n-n'|}(s^2/2) \quad (10)$$

$$\mathcal{M}(\nu) = \int_{-\infty}^{\infty} dz [\varphi_m^e(z)]^2 \eta_{\nu}(z), \quad (11)$$

\mathbf{s} is the 2D vector, $s = \sqrt{s_x^2 + s_y^2}$, $\phi = \arctg(s_y/s_x)$, $L_m^n(t)$ is the Laguerre polynomial. Without taking into account the phonon dispersion [1] the following expression for the energy E of the polaron A has been obtained:

$$E - \Sigma_1 - \frac{\sum_{\nu} |U(\nu)|^2}{E - \Sigma_0 - \hbar\omega_{LO}} = 0. \quad (12)$$

Here $U(\nu) = U_{1,0}(\nu)$. Applying (10) and (11), one obtains

$$|U(\nu)|^2 = |C_\nu|^2 (a_H^2 q_\perp^2 / 2) \exp(-a_H^2 q_\perp^2 / 2) |\mathcal{M}(\nu)|^2. \quad (13)$$

Resolving Eq. (12), one obtains

$$E_p = \frac{1}{2}(\Sigma_1 + \Sigma_0 + \hbar\omega_{LO}) \pm \sqrt{\frac{1}{4}(\Sigma_1 - \Sigma_0 - \hbar\omega_{LO})^2 + \sum_\nu |U(\nu)|^2}, \quad (14)$$

where the index p designates the magnetopolaron energy levels: $p = a$ corresponds to the upper level (the sign minus in the RHS of Eq. (14)), $p = b$ corresponds to the lower level (the sign plus in the RHS of Eq. (14)). The solution Eq. (14) is right in the resonance vicinity Eq. (3). In the resonance sharply

$$E_p^{res} = \Sigma_1 \pm \sqrt{\sum_\nu |U(\nu)|^2}, \quad (15)$$

thus, the polaron splitting equals

$$\Delta E^{res} = E_a^{res} - E_b^{res} = 2\sqrt{\sum_\nu |U(\nu)|^2}. \quad (16)$$

The magnetopolaron A wave functions for $p = a, b$ have been obtained in [23].

Taking into account the phonon dispersion and applying the method of [19], we obtain

$$E - \Sigma_1 - \sum_{\mathbf{q}_\perp, j} \frac{|U(\mathbf{q}_\perp, j)|^2}{E - \Sigma_0 - \hbar\omega_j(\mathbf{q}_\perp)} = 0 \quad (17)$$

instead of Eq. (12).

Let us consider a quite wide QW, where an approximation of interaction with the bulk phonons is applicable. We determine the phonon dispersion as following (an anisotropy of the phonon energy spectrum is neglected)

$$\omega_{LO}(\mathbf{q}) = \omega_{LO} - \Delta\omega_{LO}(q), \quad \Delta\omega_{LO}(q=0) = 0. \quad (18)$$

Then Eq. (17) takes the view

$$F(E) \equiv E - \Sigma_1 - \sum_{\mathbf{q}} \frac{|U(\mathbf{q}_\perp, q_z)|^2}{E - \Sigma_0 - \hbar\omega_{LO} + \hbar\Delta\omega_{LO}(q)} = 0, \quad (19)$$

and, according to Eqs. (7), (11), (13),

$$|U(\mathbf{q}_\perp, q_z)|^2 = \hbar\omega_{LO}^2 \frac{4\pi\alpha l}{V_0 q^2} \times (a_H^2 q_\perp^2 / 2) \exp(-a_H^2 q_\perp^2 / 2) |\mathcal{M}(q_z)|^2; \quad (20)$$

$$\mathcal{M}(q_z) = \int_{-\infty}^{\infty} dz [\varphi_m^e(z)]^2 \exp(iq_z z). \quad (21)$$

The function $F(E)$ has been calculated for the case of the square-law dispersion

$$\Delta\omega_{LO}(q) = cq^2 \quad (22)$$

and under the resonance condition of Eq. (3).

The integral had been taken as a main value for those energies E , when the denominator could equal 0. The function $F(E)$ is represented in Fig. 1 in the dispersion absence for the cases: $c = 0$, (the curve 1), $c/gl^2 = 0.04$ (the curve 2), $c/gl^2 = 0.2$ (the curve 3), $g = \alpha^{1/2}\hbar\omega_{LO}$. The curves 2 and 3 cross over the abscissa axis in the points E'_b , E_c and E'_a , obtained with taking into account the LO phonon dispersion. The crossover points

$$E_b = \Sigma_1 - \sqrt{\sum_\nu |U(\nu)|^2}, \quad E_a = \Sigma_1 + \sqrt{\sum_\nu |U(\nu)|^2}$$

correspond to the theory without the LO phonon dispersion. The differences $E_a - E'_a$ and $E'_b - E_b$ increase with growing of the dispersion parameter as it is seen in Fig. 1. The small shifts of the polaron levels correspond to the weak dispersion. The third crossover point (to which the energy E_c corresponds) appears only with taking into account the phonon dispersion. A discussion of the last result follows below.

The phonon dispersion may lead to the additional contributions into the inverse lifetimes of the polaron states [‡]. That can be explained with the help of the schematic Fig. 2, where the energy levels E'_b , E_c and E'_a are represented together with the schematic curves, depicting the dependence of the value $\Sigma_0 + \hbar\omega_{LO}(q)$ on of the 3D phonon wave vector under the resonant condition of Eq. (3). Fig. 2b corresponds to the larger phonon dispersion, than Fig. 2a. In Fig. 2b the curve $\Sigma_0 + \hbar\omega_{LO}(q)$ does not cross over the energy levels E'_a and E'_b .

That means, that the denominators $E'_a - \Sigma_0 - \hbar\omega_{LO}(q)$ and $E'_b - \Sigma_0 - \hbar\omega_{LO}(q)$ in the LHS of Eq. (19) do not equal 0 and the real solutions E'_a and E'_b are precise ones.

Applying the method of [19] one can show, that the magnetopolaron wave functions, corresponding to the precise solutions E'_a and E'_b , in the resonance vicinity of Eq. (3) have the view

$$\begin{aligned} \Theta_{p, k_y} |0\rangle &= \left[1 + \sum_\nu \frac{|U(\nu)|^2}{E'_p - \Sigma_0 - \hbar\omega_{LO}(\nu)} \right]^{-1/2} \\ &\times \left[\Psi_{1, k_y, m}^e + \sum_\nu \frac{\exp[ia_H^2 q_x (k_y - q_y / 2)]}{E'_p - \Sigma_0 - \hbar\omega_{LO}(\nu)} \right. \\ &\times U^*(\nu) \Psi_{0, k_y - q_y, m}^e b_\nu^+ \left. \right] |0\rangle, \end{aligned} \quad (23)$$

where $|0\rangle$ is the phonon vacuum wave function; p equals a or b . The functions of Eq.(24) distinguish on the corresponding functions without the phonon dispersion only by the substitution E_p by E'_p and ω_{LO} by $\omega_{LO}(\nu)$ [19]. The wave functions are orthogonal and normalized, i. e.

$$\int d^3r < 0 | \Theta_{p'k'_y}^+ \Theta_{pk_y} | 0 > = \delta_{p,p'} \delta_{k_y,k'_y}. \quad (24)$$

The orthogonalization of the wave functions Eq. (23) with indexes a and b can be checked easily if one takes into account the interrelation

$$\sum_{\nu} \frac{|U(\nu)|^2}{[E'_a - \Sigma_0 - \hbar\omega_{LO}(\nu)][E'_b - \Sigma_0 - \hbar\omega_{LO}(\nu)]} = -1, \quad (25)$$

which can be obtained, if in the LHS of Eq. (19) one substitutes E'_a , afterwards E'_b and subtracts the second expression from the first one.

As far as the energy E_c value is concerned, it is seen in Fig. 2, that the curve $\Sigma_0 + \hbar\omega_{LO}(q)$ crosses over always with the energy level E_c , because this level is as closer to the energy $\Sigma_1 = \Sigma_0 + \hbar\omega_{LO}$, as the phonon dispersion is weaker. That means that the denominator $E_c - \Sigma_0 - \hbar\omega_{LO}(q)$ in the LHS of Eq. (19) equals 0 at some absolute value q_c of the phonon wave vector. Consequently, the real value E_c is not a precise solution of Eq. (19).

In Fig. 2b the curve $\Sigma_0 + \hbar\omega_{LO}(q)$ crosses over not only the energy level E_c , but the lower polaron level E'_b also. It follows from this fact, that only real solution E'_a is precise one, but the solutions, corresponding to the energy levels a and b , must contain some imaginary parts. That means that the states a and b have the finite lifetimes, which we designate as γ_c^{-1} and γ_b^{-1} .

Let us try to calculate γ_c and γ_b . We have to generalize Eq. (19) so, that it could admit the complex solutions. The generalization suppose the substitution of the desirable energy E by $E + i\delta$, where $\delta \rightarrow +0$. Then some imaginary term will appear in Eq. (19), connected with the circuition of the integrand pole (in fact, the function $F(E + i\delta)$ is a denominator of an one-particle retarded electron Green function). Let us suppose, that the inverse lifetime γ_p of the state p is very small. Then, adopting $\tilde{E}_p = E'_p - i\hbar\gamma_p/2$, where E'_p is the real value, and applying a decomposition on the small value γ_p , one obtains from Eq. (19) in a zero approximation

$$E'_p - \Sigma_1 - Re \sum_{\mathbf{q}} \frac{|U(\mathbf{q}_{\perp}, q_z)|^2}{E'_p - \Sigma_0 - \hbar\omega_{LO}(q) + i\delta} = 0, \quad (26)$$

where $\delta \rightarrow +0$, and in the next approximation one obtains

$$\begin{aligned} \gamma_p &= -2Im \frac{1}{\hbar} \sum_{\mathbf{q}} \frac{|U(\mathbf{q}_{\perp}, q_z)|^2}{E'_p - \Sigma_0 - \hbar\omega_{LO}(q) + i\delta} = \\ &= \frac{2\pi}{\hbar} \sum_{\mathbf{q}} |U(\mathbf{q}_{\perp}, q_z)|^2 \delta[E'_p - \Sigma_0 - \hbar\omega_{LO}(q)]. \end{aligned} \quad (27)$$

Having the values γ_p , we can find, if they are small indeed. Thus, we can check, if the method, descending

to the Eqs. (26), (27), is applicable indeed. We will see below, that the method is applicable to the energy level b at the weak phonon dispersion, but inapplicable to the energy level c .

Let us substitute Eq. (20) into the RHS of Eq. (27) and use the dispersion Eq. (22). In the calculations of the function $\mathcal{M}(q_z)$ we use the wave functions

$$\varphi_m^e(z) = (2/d)^{1/2} \sin(m\pi z/d), \quad 0 < z < d \quad (28)$$

(and $\varphi_m^e(z) = 0$ outside this interval), corresponding to the QW with the infinite barriers. One obtains

$$|\mathcal{M}(q_z)|^2 \equiv f_m(Q) = \frac{2(2\pi m)^4 (1 - \cos Q)}{Q^2 [Q^2 - (2\pi m)^2]^2}, \quad (29)$$

where $Q = q_z d$, m is a number of the size-quantized energy level. Integrating the RHS of Eq. (27) on q_{\perp} with the help of the δ -function, let us represent γ_p as an integral on the variable Q

$$\gamma_p = \frac{2\alpha \hbar \omega_{LO}^2}{d(\Sigma_0 + \hbar\omega_{LO} - E'_p)} \int_0^{q_p d} dQ e^{-x} x f_m(Q), \quad (30)$$

where E'_p is the p polaron level energy, calculated, according to Eq. (26), with taking into account the phonon dispersion,

$$q_p = \sqrt{\frac{\Sigma_0 + \hbar\omega_{LO} - E'_p}{c}}, \quad (31)$$

$$x = \frac{a_H^2 q_p^2}{2} - \frac{Q^2}{\beta_0^2}, \quad (32)$$

$$\beta_0 = \frac{\sqrt{2}d}{a_H}. \quad (33)$$

If the condition of Eq. (3) is satisfied, $\beta_0 = d/l$. At any value m

$$\int_0^{\infty} dQ f_m(Q) = 3\pi/2. \quad (34)$$

Therefore the integral in the RHS of Eq. (30) is always smaller than $3\pi/2$. If the dispersion is very weak

$$dq_p \gg 1, \quad a_H q_p \gg 1, \quad (35)$$

the integral

$$\int_0^{q_p d} dQ e^{-x} x f_m(Q) \ll 1. \quad (36)$$

That means, that for the energy level $p = b$ the value $\gamma_p \rightarrow 0$ when the dispersion parameter $c \rightarrow 0$, because the value $\Sigma_0 + \hbar\omega_{LO} - E'_p$, which is in the denominator of the RHS of Eq. (30), tends to $\Sigma_0 + \hbar\omega_{LO} - E_b$. In Fig. 3

the position of the energy level E'_a is represented, as well as the broadening of the energy level E'_b as a function of the dispersion parameter c . One can see, that at the small values c the broadening is small and the approximate expression Eq. (30) for γ_b is right. However, with the increasing c the value γ_b increases so strongly, that the solutions of Eqs. (26) and (27) become incorrect. For the sake of comparison let us represent the expression for the magnetopolaron splitting ΔE^{res} , which has been obtained in [23] for the wide QWs in the limit $\beta_0 \gg 2\pi m$:

$$\Delta E^{res} = \alpha^{1/2} \hbar \omega_{LO} \sqrt{6l/d}. \quad (37)$$

Comparing Eq. (30) to Eq. (37), one finds, that $\hbar \gamma_b \ll \Delta E^{res}$ at a weak dispersion.

As far as the energy level c , according to Eq. (30), its inversion lifetime γ_c increases with the dispersion decreasing. Indeed, in the denominator of the RHS of Eq. (30) there is the value $\Sigma_0 + \hbar \omega_{LO} - E_c$, which tends to 0 with decreasing the dispersion parameter and the value $\gamma_c \rightarrow \infty$. That means that the method of the sequential approximations of Eqs. (26)-(27) is inapplicable to analyse the energy level c . The question about an existence of this level is opened up to now.

III. INFLUENCE OF THE EXCITONIC EFFECT ON THE MAGNETOPOLARON ENERGY SPECTRUM.

In the previous section we have examined a magnetopolaron, which has been formed by an electron. In this section we consider some magnetopolaron-hole pair. The influence of the Coulomb forces on the energy spectrum of an electron-hole pair (EHP) is weak under conditions

$$a_{exc}^2 \gg a_H^2, \quad a_{exc} \gg d, \quad (38)$$

where $a_{exc} = \hbar^2 \varepsilon_0 / \mu e^2$ is the radius of the Wannier-Mott exciton in a magnetic field absence, $\mu = m_e m_h / (m_e + m_h)$ is the reduced effective mass. Applying the parameters $m_e/m_0 = 0.065$, $m_h/m_0 = 0.16$, $\varepsilon_0 = 12.55$, $\hbar \omega_{LO} = 0.0367$ one obtains for GaAs

$$a_{exc} = 146A, \quad a_H^{res} = 57.2A, \quad (39)$$

$a_H^{res} = \sqrt{c\hbar/(|e|H_{res})}$, H_{res} is the magnetic field, corresponding to the magnetopolaron resonance Eq. (3), m_0 is the bare electron mass. $H_{res} = 20.2T$ for GaAs. One obtains from Eq. (39), that $(a_H^{res}/a_{exc})^2 \simeq 0.154$, i. e. the first of the conditions of Eq. (38) is satisfied, however the second condition demands to consider the QWs with the widths $d \ll 146A$, i. e. more narrower than we have considered in the previous section.

The first inequality of Eq. (38) is equivalent to the following one

$$\hbar \omega_{\mu H} / 2 \gg \Delta E_{exc}, \quad (40)$$

where $\omega_{\mu H} = |e|H/\mu c$ is the cyclotron frequency, $\Delta E_{exc} = \hbar^2/\mu a_{exc}^2$ is the exciton coupling energy in a magnetic field absence.

Under condition Eq. (38) the Coulomb interaction of an electron and hole may be considered as a weak perturbation and one can calculate the first order corrections to the EHP energy according to the perturbation theory (see [1], where the 2D case has been considered).

The EHP unperturbed wave functions are chosen as the wave functions [2] with the index \mathcal{K}_\perp and indexes n and n' , corresponding to the relative motion of the electrons and holes [26]. Let us note, that the indexes n and n' are connected single-valuedly with the Landau quantum numbers n_e and n_h of electrons and holes, respectively: at $n_e > n_h$ $n = n_h$, $n' = n_h - n_e < 0$, but if $n_e < n_h$, then $n = n_e$, $n' = n_h - n_e > 0$. The EHPs with taking into account the Coulomb forces, which can be called as excitons, are characterized by the same sets of indexes \mathcal{K}_\perp, n, n' and $\mathcal{K}_\perp, n_e, n_h$. That has been shown in [1] that the corrections to the EHP energy due to the Coulomb interaction, depend on the inplane quasi-momentum $\hbar \mathcal{K}_\perp$ in the QW plane, i. e. the Coulomb forces lead to the exciton dispersion.

The energy corrections due to the excitonic effect may be represented as two parts. The first depends only on the indexes n_e and n_h and corresponds to $\mathcal{K}_\perp = 0$.

The second (the rest part) depends on $\mathcal{K}_\perp, n_e, n_h$ and describes the exciton dispersion. The exciton energy, characterized by the quasi-wave vector \mathcal{K}_\perp , and consisting of the electron with the indexes n_e and m_e and the hole with the indexes n_h and m_h , where $m_e(m_h)$ is the number of the size-quantized energy level, is equal

$$\begin{aligned} \mathcal{E}_{n_e, n_h, m_e, m_h}(\mathcal{K}_\perp) = & E_g + \varepsilon_{m_e}^e + \varepsilon_{m_h}^h \\ & + (n_e + 1/2)\hbar \omega_{eH} \\ & + (n_h + 1/2)\hbar \omega_{hH} + \Delta \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp), \end{aligned} \quad (41)$$

where $\Delta \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp)$ is the Coulomb correction to the exciton energy. Let us separate the contribution at $\mathcal{K}_\perp = 0$:

$$\Delta \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp) = \Delta \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp = 0) + \Delta_1 \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp). \quad (42)$$

The exciton states with indexes $n_e, n_h, m_e, m_h, \mathcal{K}_\perp$ are described by the unperturbed (without taking into account the Coulomb forces) wave functions, which are not represented here.

Let us consider a pair, consisting of a magnetopolaron and hole. In the case of the A magnetopolaron [23] the following terms are overcrossed: the exciton with the indexes $n_e = 1, n_h = 1, m$ and exciton with indexes $n_e = 0, n_h = 0, m$ plus the phonon with the frequency ω_{LO} . We have chosen $n_h = 1, m_h = m_e = m$, because such combination may be created by light in the case

of the infinitely deep QW. If one omits the correction $\Delta_1 \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp)$, which depends on \mathcal{K}_\perp , the resonant condition becomes

$$\hbar\omega_{eH} = \hbar\omega_{LO} + \Delta\mathcal{E}_{0,1}(\mathcal{K}_\perp = 0) - \Delta\mathcal{E}_{1,1}(\mathcal{K}_\perp = 0). \quad (43)$$

Because the excitonic corrections $\Delta\mathcal{E}_{0,1}(\mathcal{K}_\perp = 0)$ and $\Delta\mathcal{E}_{1,1}(\mathcal{K}_\perp = 0)$ are different in values, the resonant condition Eq. (43) does not coincide with the resonant condition Eq. (3), which has been obtained without taking into account the Coulomb forces. Applying [19], one can obtain the equation for the energy \mathcal{E} of the magnetopolaron-hole pair

$$\mathcal{E} - \mathcal{E}_{1,1}(\mathcal{K}_\perp) - \sum_{\nu} \frac{|U(\nu)|^2}{\mathcal{E} - \mathcal{E}_{0,1}(\mathcal{K}_\perp - \mathbf{q}_\perp) - \hbar\omega_{LO}} = 0. \quad (44)$$

Inequalities of Eq. (38) and the estimates Eq. (39) lead to the hard restrictions of the QW width from above, what makes problematic (in any case for GaAs) the application of the bulk phonon approximation. Therefore the index ν in Eq. (44) includes the indexes \mathbf{q}_\perp and j , where j relates to the confined and interface phonons. Because a dispersion of any phonons is neglected in this section, the denominator in Eq. (44) does not depend on j .

Measuring the energy from the level

$$E_g + \varepsilon_m^e + \varepsilon_m^h + \frac{3}{2}\hbar\omega_{hH},$$

one obtains

$$\begin{aligned} \mathcal{E}_{1,1}(\mathcal{K}_\perp) &= \Sigma_1 + \Delta\mathcal{E}_{1,1}(\mathcal{K}_\perp), \\ \mathcal{E}_{0,1}(\mathcal{K}_\perp) &= \Sigma_0 + \Delta\mathcal{E}_{0,1}(\mathcal{K}_\perp). \end{aligned} \quad (45)$$

Obviously, that the energy corrections $\Delta_1 \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp)$ lead to the two essential results. First, the energy of the magnetopolaron-hole pair begins to depend on the value of the vector \mathcal{K}_\perp . At the normal light incidence $\Delta_1 \mathcal{E}_{n_e, n_h}(\mathcal{K}_\perp) = 0$, but the energy dependence on \mathcal{K}_\perp must appear at the oblique light incidence on the QW surface. Second, the term $\mathcal{E}_{0,1}(\mathcal{K}_\perp - \mathbf{q}_\perp)$ in the denominator \mathcal{E}_p of the LHS Eq. (44) must lead to the same qualitative results as the phonon dispersion, i. e. to the additional shifts of the energies of the upper and lower polaron levels and to the additional contributions into the inverse lifetimes of the polaron states. To obtain more precise results one have to take into account simultaneously the phonon dispersion and Coulomb forces.

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FIG. 1. The function $F(E)$, determining the position of the magnetopolaron energy levels (Eq. (19)) as a function of energy E under condition Eq. (3). $\beta_0 = d/l = 7.5$, $l = (\hbar/2m_e\omega_{LO})^{1/2}$, the size-quantization quantum number $m = 1$; $c = 0$ (curve 1), $c = 0.04gl^2$ (curve 2), $c = 0.2gl^2$ (curve 3), $g = \alpha^{1/2}\hbar\omega_{LO}$. E'_a, E'_b, E_c designate the position of the energy levels when the phonon dispersion exists; E_a, E_b designate the position of the energy levels when the phonon dispersion is absent.

FIG. 2. The schematic view of the magnetopolaron energy levels E'_a, E'_b, E_c and of the function $\Sigma_0 + \hbar\omega_{LO}(q)$. Fig. 2a corresponds to the lesser phonon dispersion than Fig. 2b.

FIG. 3. The position and broadening of the magnetopolaron energy levels under the resonant conditions due to the phonon dispersion as functions of the dispersion parameter c . $\beta_0 = d/l = 7.5$, , the size-quantization quantum number $m = 1$; $g = \alpha^{1/2}\hbar\omega_{LO}$. E'_a, E'_b designate the position of the energy levels when the phonon dispersion exists; E_a, E_b designate the position of the energy levels when the phonon dispersion is absent. γ_b is the inverse lifetime of the lower energy level due to the phonon dispersion.

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